Potentials between heavy-light mesons from lattice and inverse scattering theory *

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We extend our investigation of heavy-light meson-meson interactions to a system consisting of a heavy-light meson and the corresponding antiparticle. An effective potential is obtained from meson-antimeson Green-functions computed in a quenched simulation with staggered fermions. Comparisons with a simulation using an $O(a^2)$ tree-level and tadpole improved gauge action and a full QCD simulation show that lattice discretization errors and dynamical quarks have no drastic influence. Calculations from inverse scattering theory propose a similar shape for $K\bar{K}$ potentials.

Motivated by the success of QCD in high energy scattering, today one aim is to calculate hadron-hadron interactions directly from the field equations of QCD. In the low energy regime of QCD nonperturbative tools have to be used. This leads us to study the forces in systems of two hadrons on the lattice.

Previous lattice calculations have revealed a short range attractive potential between two identical heavy-light color singlets [1]. In this paper we present a lattice-QCD study of a heavy-light meson-antimeson system $M\bar{M}$ and compare the resulting residual interaction with $K\bar{K}$ potentials obtained from inverse scattering theory.

We define the one-meson field as a product of staggered Grassmann fields χ and $\bar{\chi}$ with a heavy and a light external flavor h and l, respectively,

$$\phi_{\vec{x}}(t) = \bar{\chi}_h(\vec{x}t)\chi_l(\vec{x}t)\,. \qquad (1)$$

The meson-antimeson fields with relative distance $\vec{r} = \vec{y} - \vec{x}$ are then constructed by

$$\Phi_{\vec{r}}(t) = V^{-1}\sum_{\vec{x}}\sum_{\vec{y}} \delta_{\vec{r},\vec{y}-\vec{x}}\phi_{\vec{x}}(t)\phi^{\dagger}_{\vec{y}}(t)\,. \qquad (2)$$

The dynamics of the $M\bar{M}$ system is contained in the time correlation matrix

$$C_{\vec{r}\vec{r}\,'}(t,t_0) = \langle\Phi^{\dagger}_{\vec{r}}(t)\Phi_{\vec{r}\,'}(t_0)\rangle - \langle\Phi^{\dagger}_{\vec{r}}(t)\rangle\langle\Phi_{\vec{r}\,'}(t_0)\rangle, \qquad (3)$$

where $\langle\ \rangle$ denotes the gauge field configuration average. On the hadronic level C is a two-point correlator of a composite local operator describing a molecule-like structure. Working out the contractions between the Grassmann fields we obtain

$$\begin{aligned} C_{\vec{r}}(t,t_0) &= V^{-2}\langle\sum_{\vec{x}} \mathrm{Tr}(G^{(h)\dagger}_{\vec{x}t,\vec{x}t_0} G_{\vec{x}t,\vec{x}t_0}) \\ &\times\ \mathrm{Tr}(G^{(h)}_{\vec{x}+\vec{r}t,\vec{x}+\vec{r}t_0} G^{\dagger}_{\vec{x}+\vec{r}t,\vec{x}+\vec{r}t_0})\rangle + \ldots \\ &= C^{(A)} + \ldots\,. \end{aligned} \qquad (4)$$

Since the heavy valence quarks are fixed in space the relative distance between the mesons is the same at the initial and final time of the propagation, $\vec{r}\,' = \vec{r}$. The heavy-quark propagator is

$$G^{(h)}_{\vec{x}t,\vec{x}t_0} = \left(\frac{1}{2m_h a}\right)^k [\Gamma_{\vec{x}4}]^k \prod_{j=1}^{k} U_{x=(\vec{x},ja),\mu=4}\,, \qquad (5)$$

where the phase factors $\Gamma_{\vec{x}4} = (-1)^{(x_1+x_2+x_3)/a}$ in the Kogut-Susskind formulation correspond to the Dirac matrices and $k = (t-t_0)/a$. In our calculations we set $m_h a = 0.5$. The propagator of the light quark is obtained from inverting the

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staggered fermion matrix with a random source estimator. A standard conjugate gradient algorithm is used.

The effective ground-state energy of the $M\bar{M}$ system $W(\vec{r})$ can be extracted from the large euclidean time behavior of C following quantum-mechanical reasoning for composite particles,

$$C_{\vec{r}}(t, t_0) \quad \simeq \quad c(\vec{r}) e^{-W(\vec{r})(t-t_0)} \,. \tag{6}$$

The residual meson-antimeson potential is

$$V(r) = W(r) - 2m \,, \tag{7}$$

where $2m$ is the mass of two free (anti)mesons.

The simulations were performed on a periodic $8^3 \times 16$ lattice. Each potential is the result of a measurement on 100 independent gauge field configurations separated by 200 sweeps. The inversion of the fermion matrix was performed with 32 random sources.

Each contribution to the correlator in (4) comprises the exchange of gluons. As a first step only the direct term $C^{(A)}$ was computed, the computation of the full correlator being the task of a subsequent work. The effective energy $W(r)$ was then extracted from the correlator $C^{(A)}$ by a four parameter (Levenberg-Marquardt) fit with the function

$$\begin{aligned} C_r^{(A)} \quad &= \quad B(r) \cosh\left[W(r)(t-8a)\right] \\ &+ \quad (-1)^{t/a} \widetilde{B}(r) \cosh[\widetilde{W}(r)(t-8a)]. \end{aligned} \tag{8}$$

The second term alternating in sign is a peculiarity of the staggered scheme. To be numerically consistent with $W(r)$ at large distances, the mass $2m$ of two noninteracting mesons was extracted from the square of the *one*-meson two-point function.

In order to investigate the influence of discretization errors and sea quarks a simulation using an improved gauge action and a full QCD simulation were also performed. The former includes planar six-link plaquettes in addition to the elementary plaquettes and is tadpole improved [2]. The resulting potentials are displayed in Fig. 1. All simulations show consistent results. The potential turns out to be short ranged and attractive. At distances $r/a \geq 1$ essentially no interaction energy can be resolved.

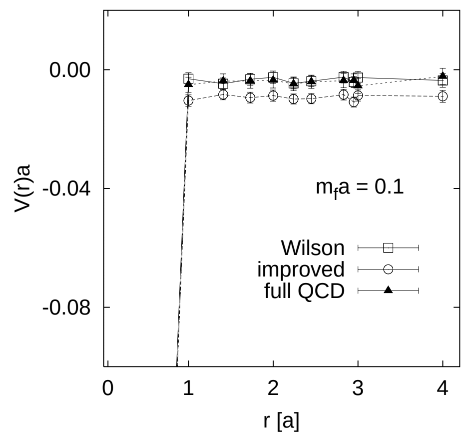

Figure 1. Heavy-light meson-antimeson potentials $V(r)a$ for quark mass $m_f a = 0.1$, from a quenched simulation using the Wilson gauge action with $\beta = 5.6$, from an improved simulation with $\beta_{pl} = 7.0$ corresponding to the same lattice constant $a \approx 0.2\,\mathrm{fm}$, and from a full QCD simulation with number of flavors $n_f = 3$ and $\beta = 5.2$.

Data from an independent simulation for a different light-quark mass $m_f a = 0.05$ were used to make a linear extrapolation to light quark mass zero. To estimate the strength of the interaction the data were matched to a Gaussian potential. The latter was then used to obtain scattering phase shifts from a Schrödinger equation with three arbitrary values of the meson mass. The results are shown in the left plots of Fig. 2.

An effective range expansion with two sets of parameters was applied to experimental data of $K\bar{K}$ s-wave scattering [3]. Using quantum inversion these analytic phase functions can be transformed into local potentials [4]. The input phase shifts and the resulting potentials are shown in the right plots of Fig. 2. We notice a good qualitative agreement between lattice and inversion potentials for short distances. The repulsive hump could not be resolved by our simulation. This feature, as it turns out, is decisive for the phase shifts and explains the lack of qualitative agreement between δ_0.

The next step in our program is to compute the full correlator C_r. A new simulation with an

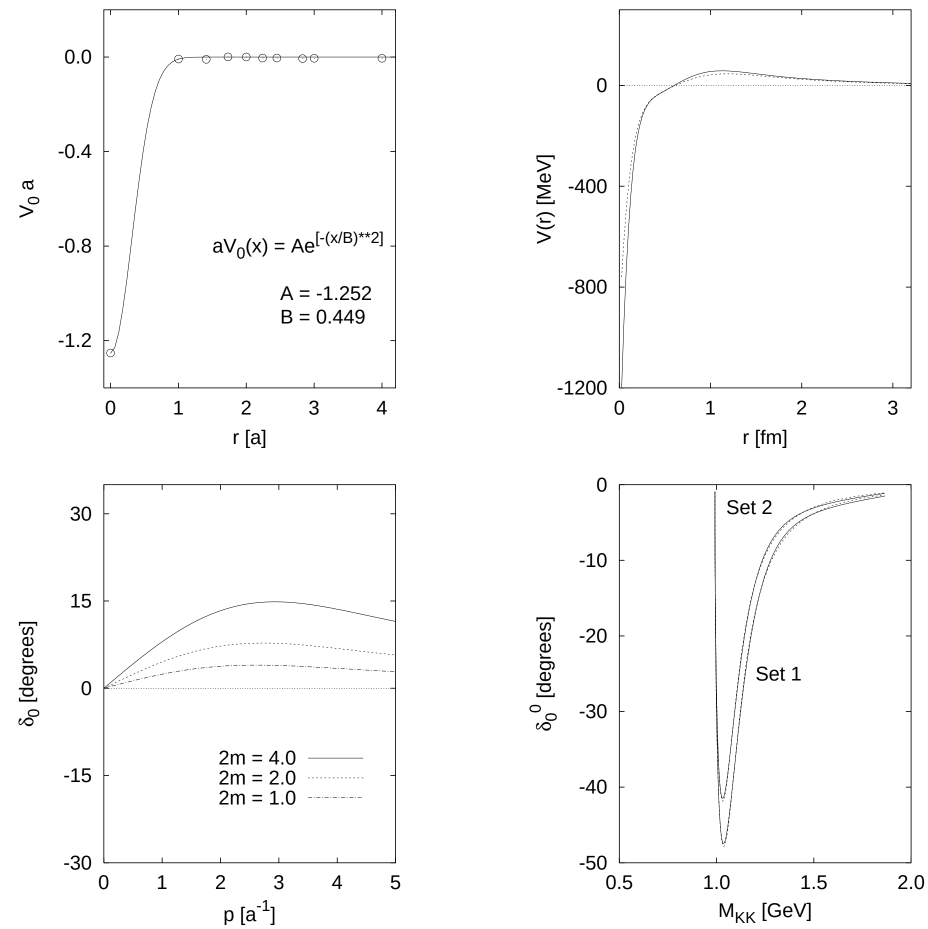

Figure 2. Potentials and phase shifts for the $M\bar{M}$ system from the lattice (left plots) and for the $K\bar{K}$ system from inverse scattering (right plots).

$O(a^2)$ tree-level and tadpole improved gauge action and Naik-type improved staggered fermionic action is in progress. An independent lattice investigation with $O(a^2)$-improved lattice action using Wilson fermions is presented in these proceedings.

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